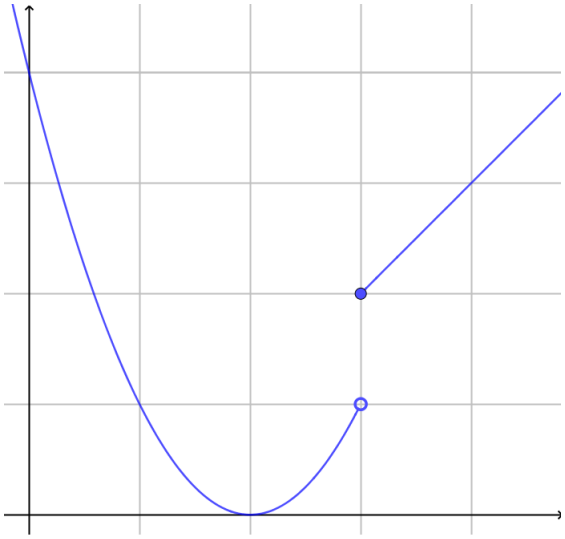
MTH 1000 Final Exam Name: _____

Time: 205 minutes, Max Points: 200 [+5 Bonus]

Show **all work clearly** and in order to receive full credit. Use of books, notes, calculators, or any other resource is **not** permitted.

1. [10 points] Consider the graph of a function $f(x)$ below.



(a) Find the x-intercept(s).

(b) Find the y-intercept(s).

(c) Is f a continuous function?

(d) Find $f(1)$.

(e) Find $f(3)$.

2. [10 points] By completing the square, find the center and radius for the circle with the equation

$$x^2 + y^2 + 4x - 6y - 3 = 0$$

3. [10 points] Find the equation of the line (in $y = mx + b$ form) that passes through the point $(1, 1)$ and

   (a) is perpendicular to the line $3x - 6y = 1$.

   (b) is parallel to the line $3x - 6y = 1$.

4. [10 points] Solve the following quadratic equations.

   (a) $x^2 + 7x + 12 = 0$

   (b) $3x^2 + 5x - 10 = 5$

5. [10 points] Consider the piecewise function $f(x) = \begin{cases} 1, & \text{if } x < -1 \\ x, & \text{if } -1 \leq x \leq 1 \\ -1, & \text{if } x > 1 \end{cases}$.

   (a) Graph the f.

   (b) State the domain and range of f.

   (c) Is the function even, odd, or neither?

6. [10 points] Find the domain, range, all intercepts, and all asymptotes (if they exist) of the function $f(x) = \frac{x-1}{x^2-9}$ and draw its graph.

7. [10 points] Let $f(x) = x^2 - 1$ and $g(x) = 4x + 2$. Find the formula, domain, and range for each of the following, if possible.

(a) $(f \circ g)(x)$

(b) $g^{-1}(x)$

8. [10 points] Solve each logarithmic equation.

(a) $\log_4(2x - 1) = 2$

(b) $\log(x + 3) = 1 - \log(x + 2)$

9. [6 points] Convert the radian measurements into degrees and determine the quadrant where each angle lies.

(a) $\frac{2\pi}{3}$

(b) 4π

10. [24 points] Fill in the table.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$0°$			
$30°$			
$45°$			
$60°$			
$90°$			
$180°$			
$270°$			
$360°$			

11. [10 points] Find the exact value of the following trigonometric functions.

(a) $\sin(300°)$

(c) $\sec\left(\frac{4\pi}{3}\right)$

12. [10 points] If $\csc\theta = 2$ and $\cos(\theta) > 0$, find the value of the other 5 trig functions at θ.

13. [10 points] Find the period, amplitude, phase shift, x-intercepts, and y-intercepts of $f(x) = 2\sin\left(2x + \frac{\pi}{2}\right) + 1$ and sketch the graph of at least two periods of the function.

14. [10 points] Evaluate the inverse trigonometric expressions.

(a) $\sin^{-1}(-1)$

(b) $\cos^{-1}\left(\frac{1}{2}\right)$

15. [10 points] Evaluate the inverse trigonometric expressions, if possible.

(a) $\sin\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

(b) $\cos(\cos^{-1}(-2))$

16. [10 points] Evaluate the trigonometric expressions.

(a) $\tan\left(\sin^{-1}\left(\frac{-\sqrt{2}}{2}\right)\right)$

(b) $\sin^{-1}(\sec(\pi))$

17. [10 points] Find all the solutions in the interval $0 \leq \theta < 2\pi$.

(a) $\sin\theta = \cos\theta$

(b) $\sin(2\theta) = \frac{1}{2}$

18. [10 points] Evaluate $\sin\left(\sec^{-1}\left(\frac{5}{4}\right) - \tan^{-1}(4)\right)$.

19. [10 points] If $\cos\theta = -\frac{3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, evaluate the following.

(a) $\sin\frac{\theta}{2}$

(b) $\cos\frac{\theta}{2}$

Bonus [+5 points] Show that $\tan(\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3) = 0$ given that

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta.}$$