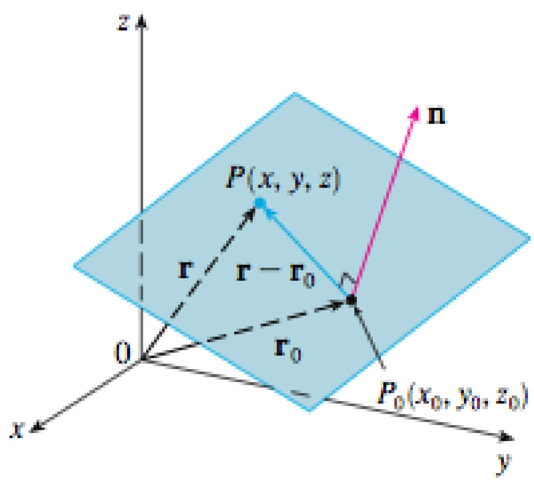
Calculus 3: Equations of Lines and Planes Notes

- 3 Ways to Represent a Line in 3D
  - Vector Equation: $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$
    * The line passes through the point $\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle$
    * The line is parallel to the vector $\mathbf{v} = \langle a, b, c \rangle$
    * a, b, and c are called the direction numbers
    * t can take the value of any real number. It is called the parameter
  - Parametric Equations: $\langle x(t), y(t), z(t) \rangle = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$
  - Symmetric Equations: $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$
- Line Segments from $\mathbf{r}_0$ to $\mathbf{r}_1$
  - Equation of a line segment: $\mathbf{r}(t) = (1-t)\mathbf{r}_0 + t\mathbf{r}_1$, $0 \leq t \leq 1$
  - Notice $\mathbf{r}(0) = \mathbf{r}_0$ and $\mathbf{r}(1) = \mathbf{r}_1$
- 3 Ways to Represent a Plane
  - Vector Equation: $\mathbf{n} \cdot (\mathbf{r} - \mathbf{r_0}) = 0$ or $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$
    * $\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle$ and $\mathbf{r} = \langle x, y, z \rangle$ are vectors on the plane
    * $\mathbf{n} = \langle a, b, c \rangle$ is a normal (i.e. orthogonal) vector to the plane



*

- Scalar Equation: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$
- Linear Equation: $ax + by + cz + d = 0$

- Distance from a Point $P_1(x_1, y_1, z_1)$ to a Plane $ax + by + cz + d = 0$
  - $D = \text{comp}_{\mathbf{n}}\mathbf{b} = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$