Algebra Review

Properties of exponents

Addition: $cx^a + dx^a = (c+d)x^a$
Subtraction: $cx^a - dx^a = (c-d)x^a$
Multiplication: $cx^a \times dx^b = (cd)x^{a+b}$
Division: $\frac{cx^a}{dx^b} = (\frac{c}{d})x^{a-b}$
Distribution: $(cd)^a = c^a d^a$
Power of a Power: $c(x^a)^b = cx^{ab}$

Trigonometry

http://web.mit.edu/wwmath/trig/identities02.html

Pythagorean and double angle identities are very helpful.

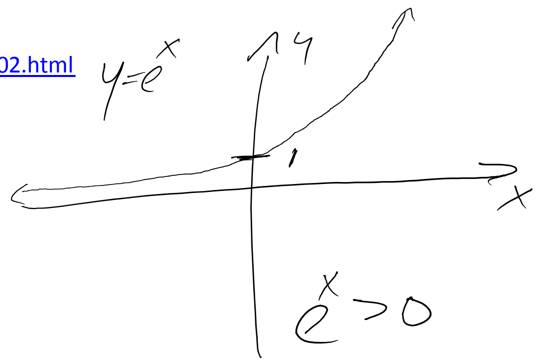
Radians = degrees * (pi/180)

Math *always* uses radians by default!

Properties of e^x

- Exponents of e work the same as any other exponents (e is just an irrational number like pi)
- $\int e^x dx = e^x + C$
- $e^{\ln x} = x$
- $\ln(e^x) = x$

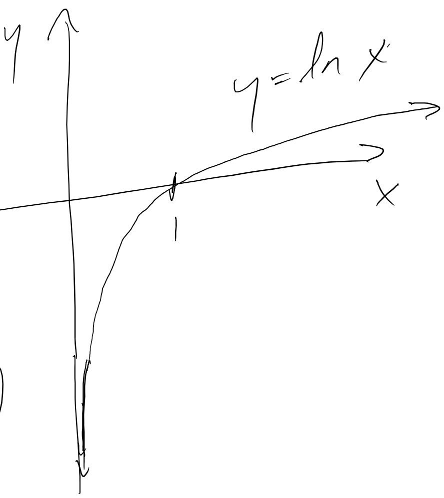
(e^x and $\ln x$ are inverses)



$e^x > 0$

Properties of ln/logs

- undefined for $x \leq 0$
- $\ln(xy) = \ln x + \ln y$
- $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$ (if $y \neq 0$)
- $\ln(x^n) = n \ln x$
- $\int \frac{1}{x} dx = \ln|x| + C$



Calculus 1 Review

Derivative Formulas

From the front cover of Zill:

1. Constant: $\frac{d}{dx}c = 0$	**2.** Constant Multiple: $\frac{d}{dx}cf(x) = c\,f'(x)$
2. Sum: $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	**4.** Product: $\frac{d}{dx}f(x)g(x) = f(x)g'(x) + g(x)f'(x)$
5. Quotient: $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$	**6.** Chain: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$
7. Power: $\frac{d}{dx}x^n = nx^{n-1}$	**8.** Power: $\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}g'(x)$

u-substitution

Explanation from FIT's Calculus 1 lab sheets:

To reduce a complicated integral into a simpler one by u-substitution, choose some part of the formula to be re-labeled as a new variable u. Then proceed to re-write the formula by replacing all the u-terms and the dx-term with a corresponding formula in terms of du. If you are successful, a simpler formula will result that should be easier to integrate:

$$\int f(x)dx \Rightarrow \text{Let } u = g(x) \Rightarrow \tfrac{du}{dx} = g'(x) \Rightarrow du = g'(x)\,dx \Rightarrow dx = \tfrac{du}{g'(x)}$$

Relabeling:

$$\int f(x)dx = \int h(u)du$$

$$\text{Old } x\text{ - Function} \Rightarrow \text{New } u\text{ - Function}$$

The process of integration by u-substitution is identical to the process for indefinite integrals *except* that when there are limits involved in the definite integral, the x-limits in the original problem must be converted to new u-limits before they can be used and plugged in:

If $\int_{x=a}^{x=b} f(x)dx$ and you make the substitution $u = g(x)$ then $\begin{matrix} x = b \Rightarrow u = g(b) \\ x = a \Rightarrow u = g(a) \end{matrix}$

and so $\int_{x=a}^{x=b} f(x)dx = \int_{u=g(a)}^{u=g(b)} h(u)du$

The possibly less clear statement of the property from Stewart's Calculus (Section 5.5):

4 THE SUBSTITUTION RULE If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du$$

Notice it's really just the inverse of the chain rule.

Calculus 2 Review

Integration by Parts (Section 7.1 in Stewart's Calculus)

If f and g are differentiable functions, we have

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx$$

Or, in simpler notation:

$$\int u\,dv = uv - \int v\,du$$

(This rule is a sort of inverse of the product rule.)

If you have a definite integral, **don't forget to use the *a* and *b* in the term before the minus sign** (otherwise, we would still have x terms after the definite integral, which is incorrect):

$$\int_a^b f(x)g'(x)\,dx = f(x)g(x)\Big]_a^b - \int_a^b g(x)f'(x)\,dx$$

The formula isn't too hard to remember, but the more difficult part is to determine which parts of the integral should be u, and which should be dv. Often a good strategy for choosing what function should be u is the LIATE rule, which says whichever comes first in this list should be *u*:

L	Logarithmic functions	ln(x), $\log_a(x)$, etc.
I	Inverse trig functions	$\sin^{-1}(x)$, $\tan^{-1}(x)$, etc.
A	Algebraic functions	3x, x^5, etc.
T	Trig functions	tan(x), cos(x), etc.
E	Exponential functions	e^x, 10^x, etc.

There are exceptions to this rule, but it is generally very helpful. Sometimes it's necessary to repeat integration by parts multiple times and/or use it after using *u*-substitution.

Partial Fraction Decomposition (Section 7.4 in Stewart's Calculus)

If substitutions do not offer a clear path to a solution, partial fraction decomposition is usually the next choice. The Wikipedia page on this is significantly better than what I can provide, and in particular, these sections are helpful:

http://en.wikipedia.org/wiki/Partial_fraction_decomposition#Procedure

http://en.wikipedia.org/wiki/Partial_fraction_decomposition#Illustration

http://en.wikipedia.org/wiki/Partial_fraction_decomposition#Examples

Section 7.5 of Stewart's Calculus provides an overview of integration strategy that would also be a great place to get some general tips on how to approach different types of integrals. The book is available in the MAC if you don't have it or cannot borrow one (other calculus books have sections on the same topics).

Khan Academy has helpful short videos on all of these topics on YouTube as well.

Derivative/Integral Pairs Involving Trig Functions

From the cover of Zill:

Trigonometric:

9. $\frac{d}{dx}\sin x = \cos x$

10. $\frac{d}{dx}\cos x = -\sin x$

11. $\frac{d}{dx}\tan x = \sec^2 x$

12. $\frac{d}{dx}\cot x = -\csc^2 x$

13. $\frac{d}{dx}\sec x = \sec x \tan x$

14. $\frac{d}{dx}\csc x = -\csc x \cot x$

Inverse trigonometric:

15. $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

16. $\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$

17. $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$

18. $\frac{d}{dx}\cot^{-1} x = -\frac{1}{1+x^2}$

19. $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$

20. $\frac{d}{dx}\csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$

1. $\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \; n \neq -1$

2. $\int \frac{1}{u} du = \ln|u| + C$

39. $\int \frac{1}{\sqrt{a^2-u^2}} du = \sin^{-1}\frac{u}{a} + C$

40. $\int \frac{1}{\sqrt{a^2+u^2}} du = \ln\left|u + \sqrt{a^2+u^2}\right| + C$

41. $\int \sqrt{a^2-u^2}\, du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$

42. $\int \sqrt{a^2+u^2}\, du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln\left|u + \sqrt{a^2+u^2}\right| + C$

43. $\int \frac{1}{a^2+u^2} du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

44. $\int \frac{1}{a^2-u^2} du = \frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right| + C$